كلية الحاسبات والذكاء الإصطناعي

SC311
Modeling and Simulation

Lecture 04

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Spring 2023

Chapter 3: Queueing Simulation

- Introduction.
- Description of Queuing System.
- Simulating a Single-Server Queue.
  - ✓ Example.
- Simulating a Queue with Multi-Servers.
  - ✓ Example.

Sim. a Single-Server Queue (9/14)

Simulation Table for Queueing Problem (20 customers)

	Simulation System					Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1	15	3	18	21	3	6	0
5	8	23	2	23	25	0	2	2
6	3	26	4	26	30	0	4	1
7	8	34	5	34	39	0	5	4
8	7	41	4	41	45	0		2
9	2	43	5	45	50			
10	3	46	3	50	53			
11	1	47	3	53	56			
12	1	48	5	56	61			
13	5	53	4	61	65			
14	6	59	1	65	66			0
15	3	62	5	66	71	4	9	0
16	8	70	4	71	75	1	5	0
17	1	71	3	75	78	4	7	0
18	2	73	3	78	81	5	8	0
19	4	77	2	81	83	4	6	0
20	5	82	3	83	86	1	4	0
Total	82		68			56	124	18

Total Run Time of Simulation

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (1/7):

$$\text{average waiting time (minutes)} = \frac{\text{total time customers wait in queue (minutes)}}{\text{total numbers of customers}}$$

$$= \frac{56}{20} = 2.8 \text{ minutes}$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (2/7):

$$\text{probability (wait)} = \frac{\text{number of customers who wait}}{\text{total number of customers}}$$

$$= \frac{13}{20} = 0.65$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (3/7):

$$\text{probability of idle server} = \frac{\text{total idle time of server (minutes)}}{\text{total run time of simulation (minutes)}}$$

$$= \frac{18}{86} = 0.21$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (4/7):

$$\text{average service time (minutes)} = \frac{\text{total service time (minutes)}}{\text{total number of customers}}$$

$$= \frac{68}{20} = 3.4 \text{ minutes}$$

This result can be compared with the expected service time by finding the mean of the service-time distribution using the equation:

$$E(S) = \sum_{s=0}^{\infty} sp(s)$$

$$= 1(0.10) + 2(0.20) + 3(0.30) + 4(0.25) + 5(0.10) + 6(0.05)$$

$$= 3.2 \text{ minutes}$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (5/7):

$$\text{average time between arrivals (minutes)} = \frac{\text{sum of all times between arrivals (minutes)}}{\text{number of arrivals} - 1}$$

$$= \frac{82}{19} = 4.3 \text{ minutes}$$

The mean is given

$$E(A) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5 \text{ minutes}$$

The expected time between arrivals is slightly higher than the average. However, as the simulation becomes longer, the average value of the time between arrivals will approach the theoretical mean, $E(A)$.

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (6/7):

$$\text{Average waiting time of those who wait (minutes)} = \frac{\text{total time customers wait in queue (minutes)}}{\text{total number of customers who wait}}$$

$$= \frac{56}{13} = 4.3 \text{ minutes}$$

Sim. a Single-Server Queue (10/14)

Some of the findings from the simulation are as follows (7/7):

$$\text{average time customer spends in the system (minutes)} = \frac{\text{total time customers spend in the system (minutes)}}{\text{total number of customers}}$$

$$= \frac{124}{20} = 6.2 \text{ minutes}$$

The second way of computing this same result is to realize that the following relationship must hold:

$$= 2.8 + 3.4 = 6.2 \text{ minutes}$$

$$\begin{array}{c} \text{average time customer spends in the system (minutes)} \\ = \\ \text{average time customer spends waiting in the queue (minutes)} \\ + \\ \text{average time customer spends in service (minutes)} \end{array}$$

Sim. a Single-Server Queue (11/14)

- Our objective is to analyze the system by simulating the arrival and service of ***100 customers*** and to compute a variety of typical measures of performance for queueing models.
  - In actuality, 100 customers may be too small a sample size to draw reliable conclusions. Depending on our objectives, the accuracy of the results may be enhanced by increasing the sample size (number of customers), or by running multiple trials (or replications).

Sim. a Single-Server Queue (12/14)

- Our objective is to analyze the system by simulating the arrival and service of ***100 customers*** and to compute a variety of typical measures of performance for queueing models.
  - A second issue is that of initial conditions. A simulation of a grocery store that starts with an empty system may or may not be realistic unless the intention is to model the system from startup or to model until steady-state operation is reached. Here, to keep calculations simple, the starting conditions are an empty grocery, and any concerns are overlooked.

Sim. a Single-Server Queue (13/14)

Simulation Table for Queueing Problem (100 customers)

Simulation System | Performance Measure

Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	-	0	4	0	4	0	4	0
2	1	1	2	4	6	3	5	0
3	1	2	5	6	11	4	9	0
4	6	8	4	11	15	3	7	0
5	3	11	1	15	16	4	5	0
6	7	18	5	18	23	0	5	2
...								
100	5	415	2	416	418	1	3	0
Total	415		317			174	491	101

Sim. a Single-Server Queue (14/14)

Some of the findings from the simulation are as follows (1/4):

Average waiting time	$\overline{w} = \frac{\sum \text{Waiting time in queue}}{\text{Number of customers}} = \frac{174}{100} = 1.74 \text{ min}$
Probability that a customer has to wait	$p(\text{wait}) = \frac{\text{Number of customer who wait}}{\text{Number of customers}} = \frac{46}{100} = 0.46$
Proportion of server idle time	$p(\text{idle server}) = \frac{\sum \text{Idle time of server}}{\text{Simulation run time}} = \frac{101}{418} = 0.24$
Average service time	$\overline{s} = \frac{\sum \text{Service time}}{\text{Number of customers}} = \frac{317}{100} = 3.17 \text{ min}$ $E(s) = \sum_{s=0}^{\infty} s \cdot p(s) = 0.1 \cdot 10 + 0.2 \cdot 20 + \cdots + 0.05 \cdot 6 = 3.2 \text{ min}$

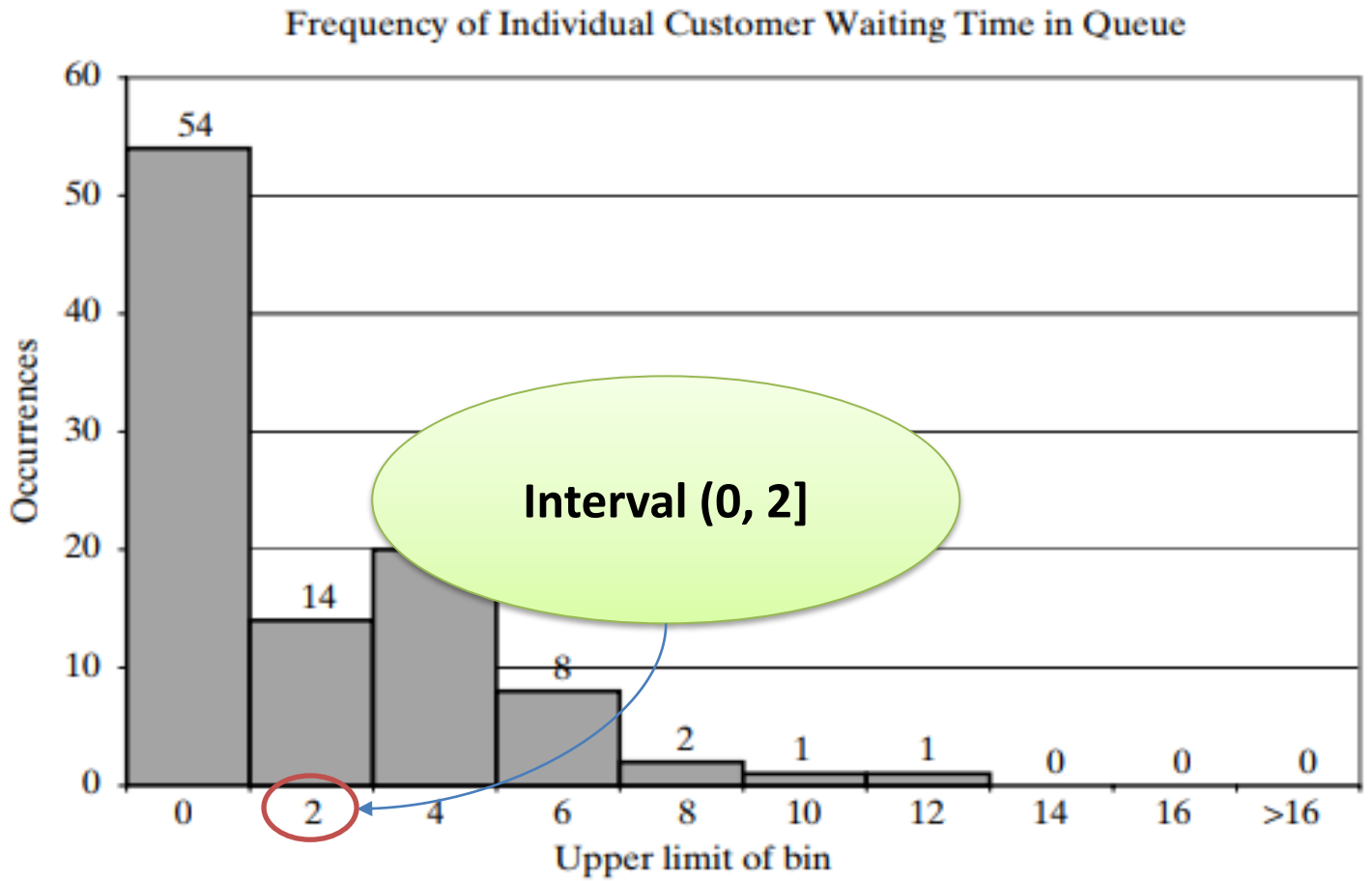
Sim. a Single-Server Queue (14/14)

Some of the findings from the simulation are as follows (1/4):

Average time between arrivals	$\bar{\lambda} = \frac{\sum \text{Times between arrivals}}{\text{Number of arrivals} - 1} = \frac{415}{99} = 4.19 \text{ min}$ $E(\lambda) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5 \text{ min}$
Average waiting time of those who wait	$\bar{w}_{waited} = \frac{\sum \text{Waiting time in queue}}{\text{Number of customers that wait}} = \frac{174}{54} = 3.22 \text{ min}$
Average time a customer spends in system	$\bar{t} = \frac{\sum \text{Time customers spend in system}}{\text{Number of customers}} = \frac{491}{100} = 4.91 \text{ min}$ $\bar{t} = \bar{w} + \bar{s} = 1.74 + 3.17 = 4.91 \text{ min}$

Sim. a Single-Server Queue (14/14)

Some of the findings from the simulation are as follows (2/4):



Sim. a Single-Server Queue (14/14)

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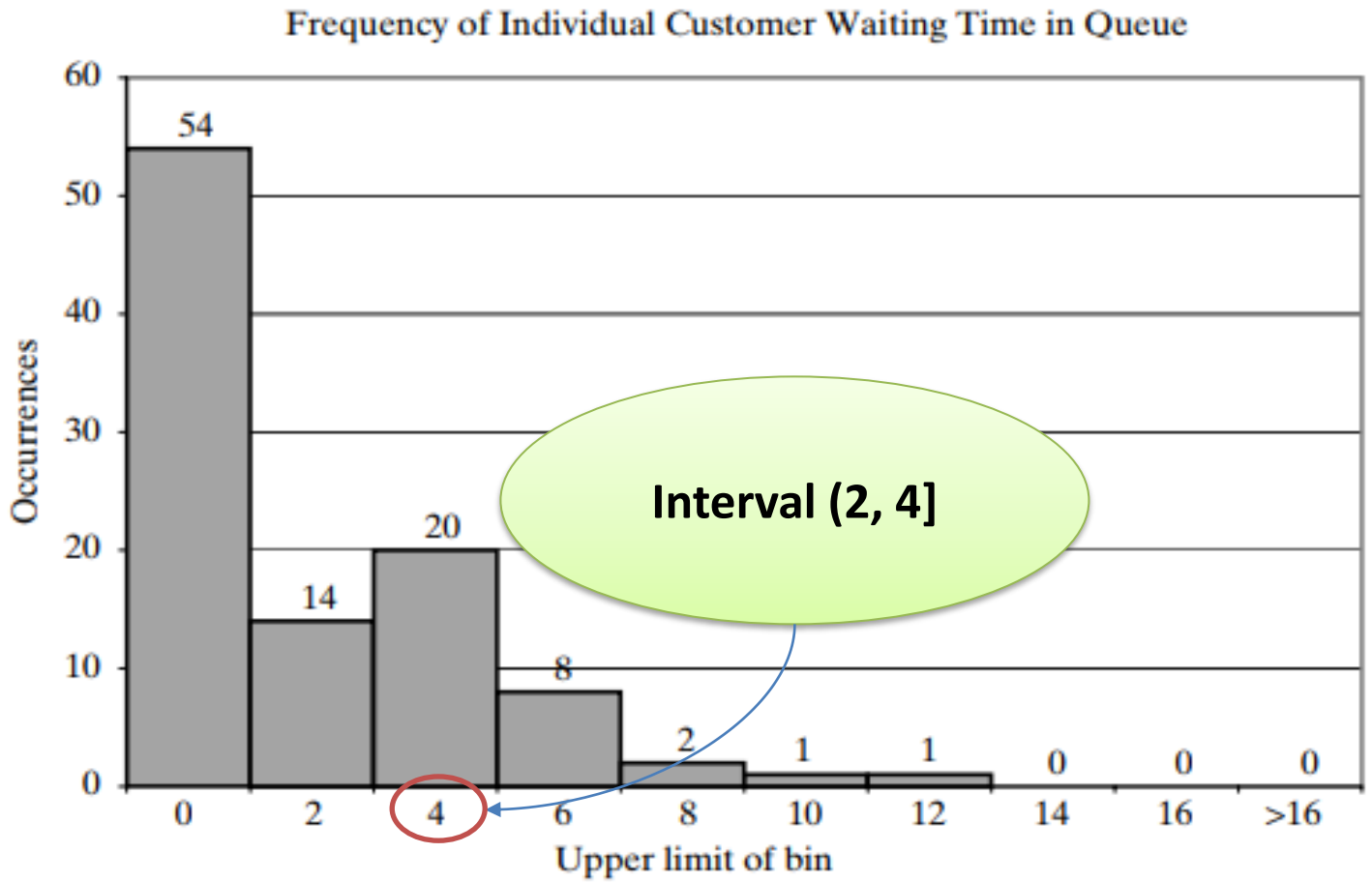

Sim. a Single-Server Queue (14/14)

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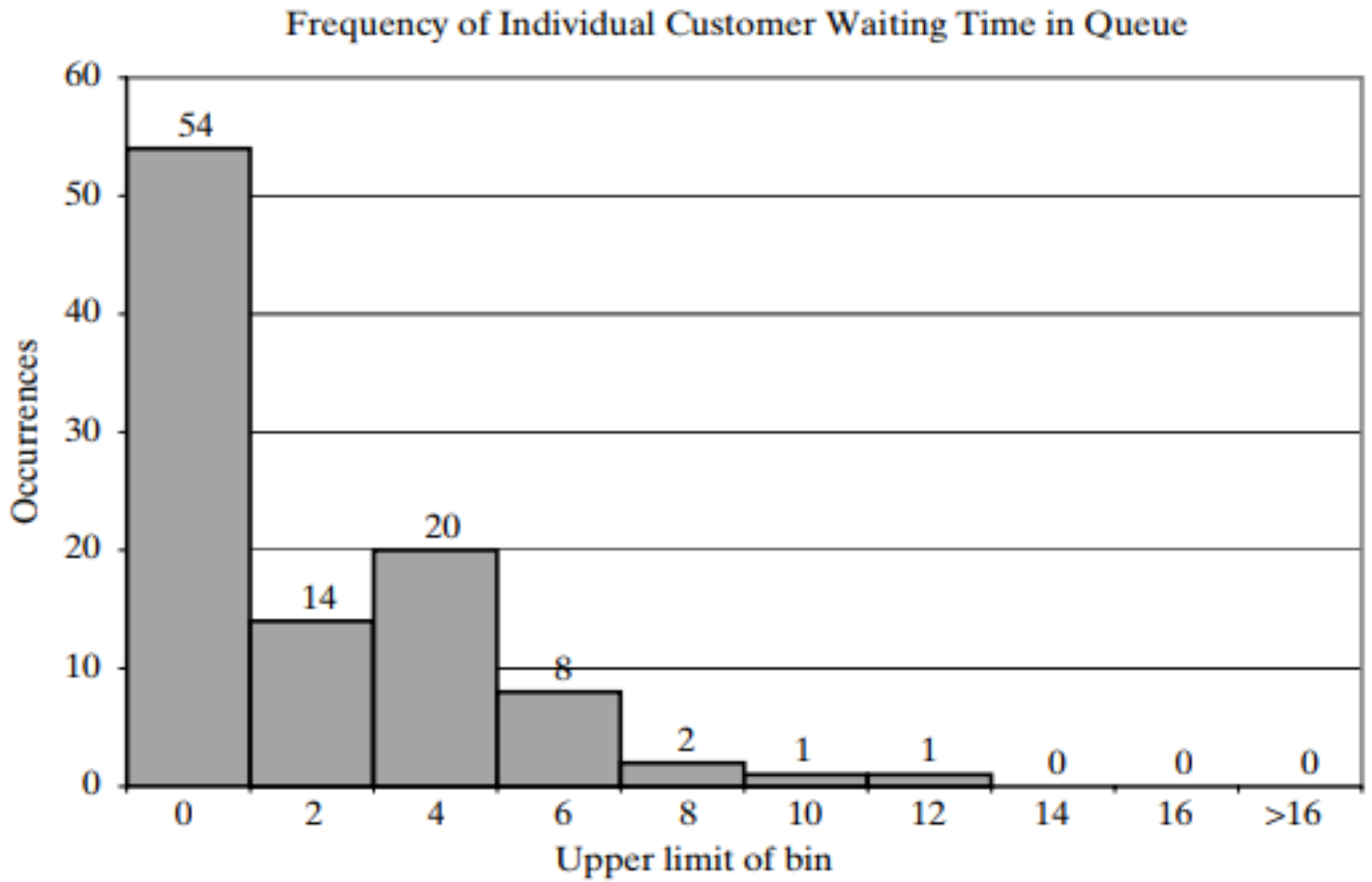


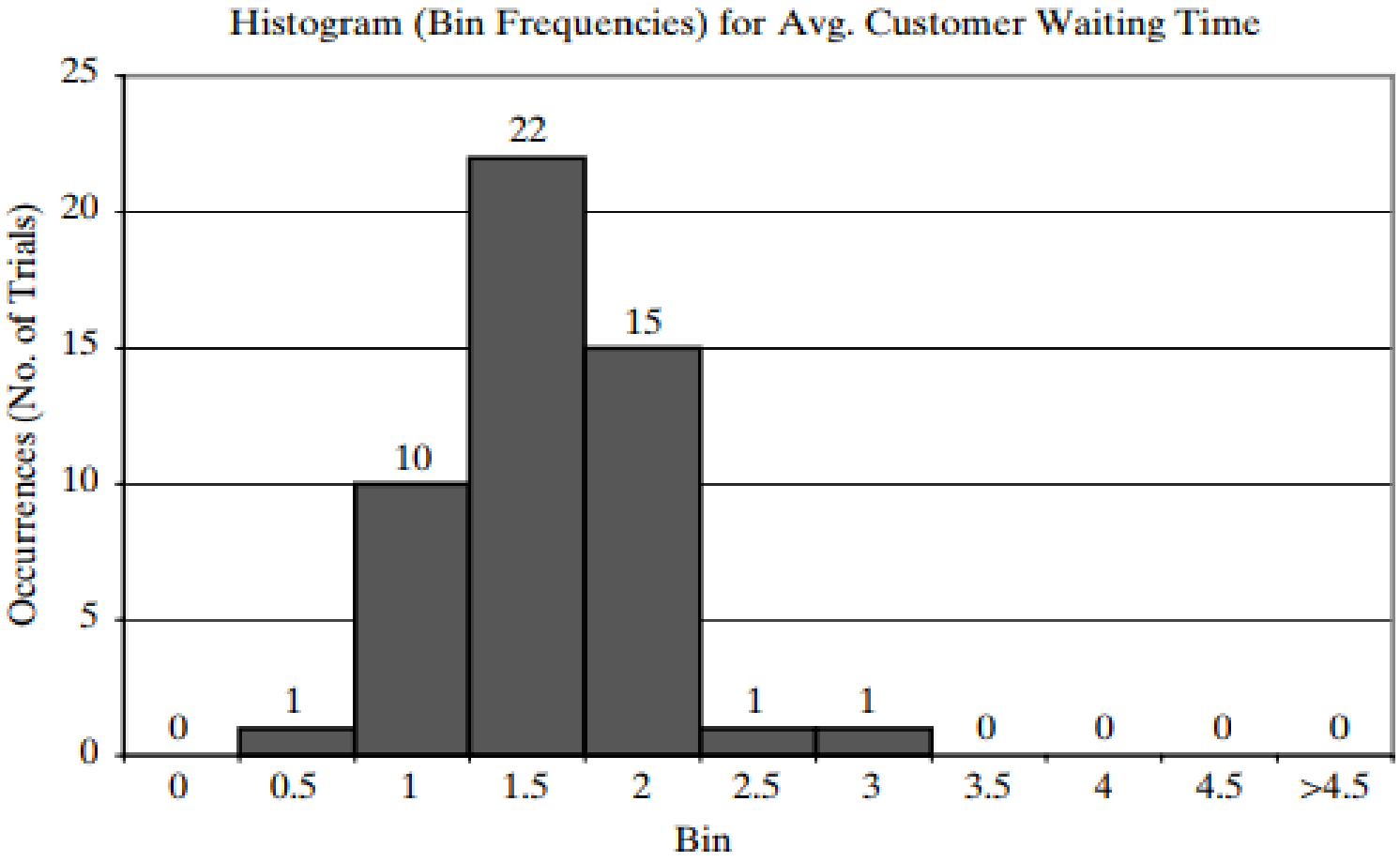
Sim. a Single-Server Queue (14/14)

Some of the findings from the simulation are as follows (3/4):

- We are now interested in running an experiment to address the question of *how the average time in queue for the first 100 customers varies from day to day*.
- Running once with 100 customers corresponds to one day. Running it 50 times corresponds to 50 days, where each trial represents one day.
- The overall average waiting time over 50 trials was 1.32 minutes.

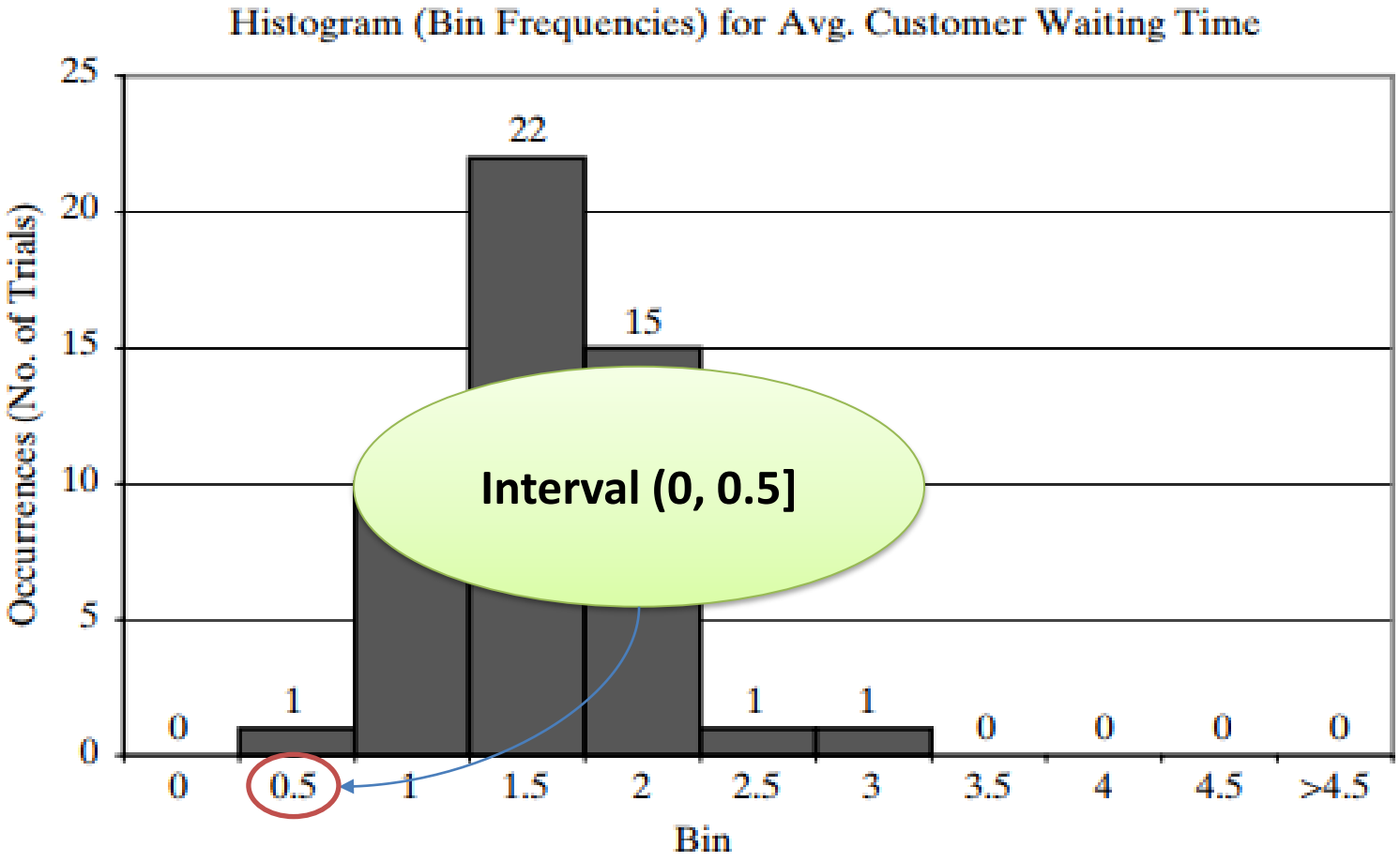
Sim. a Single-Server Queue (14/14)

Some of the findings from the simulation are as follows (4/4):



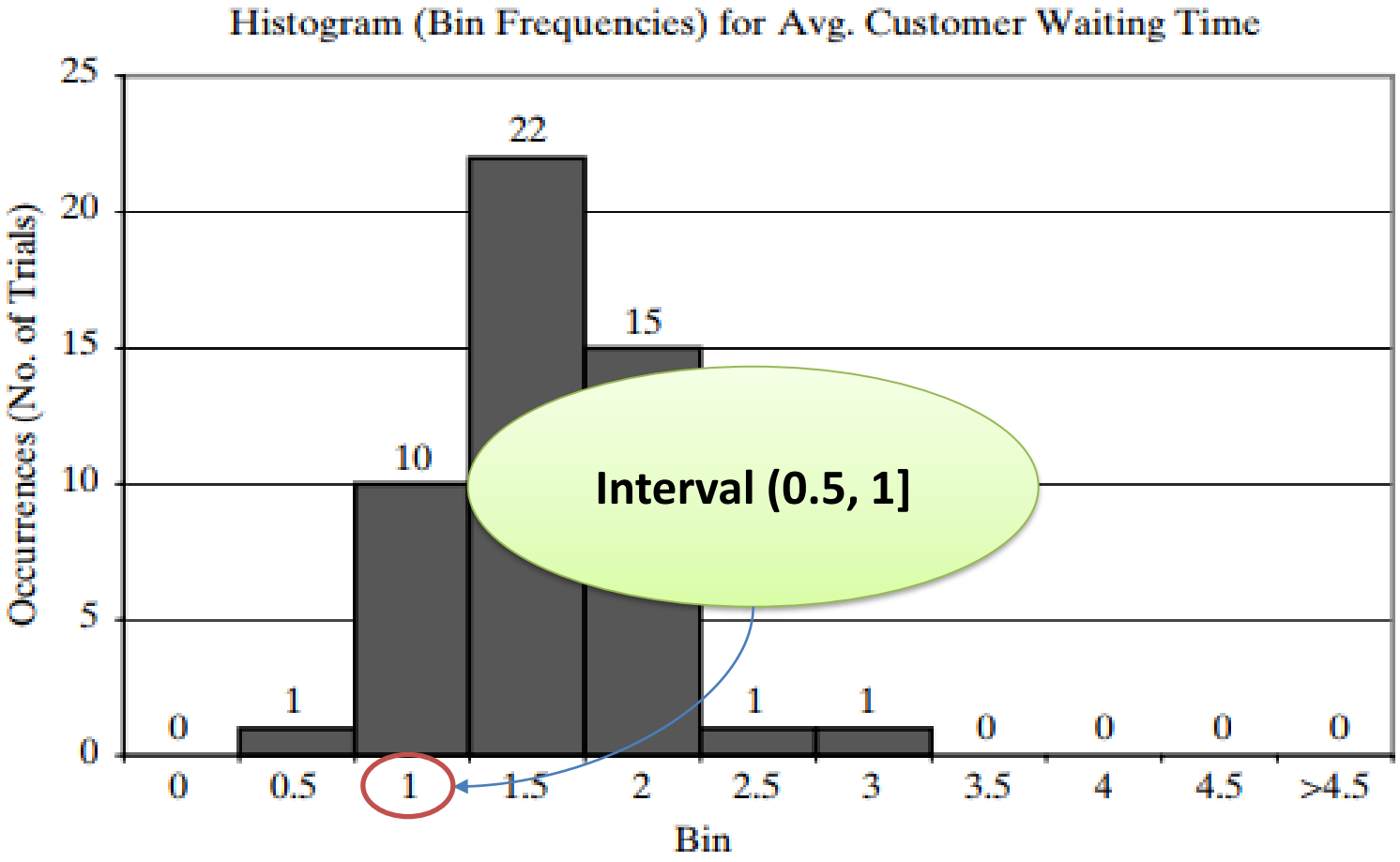
Sim. a Single-Server Queue (14/14)

Some of the findings from the simulation are as follows (4/4):



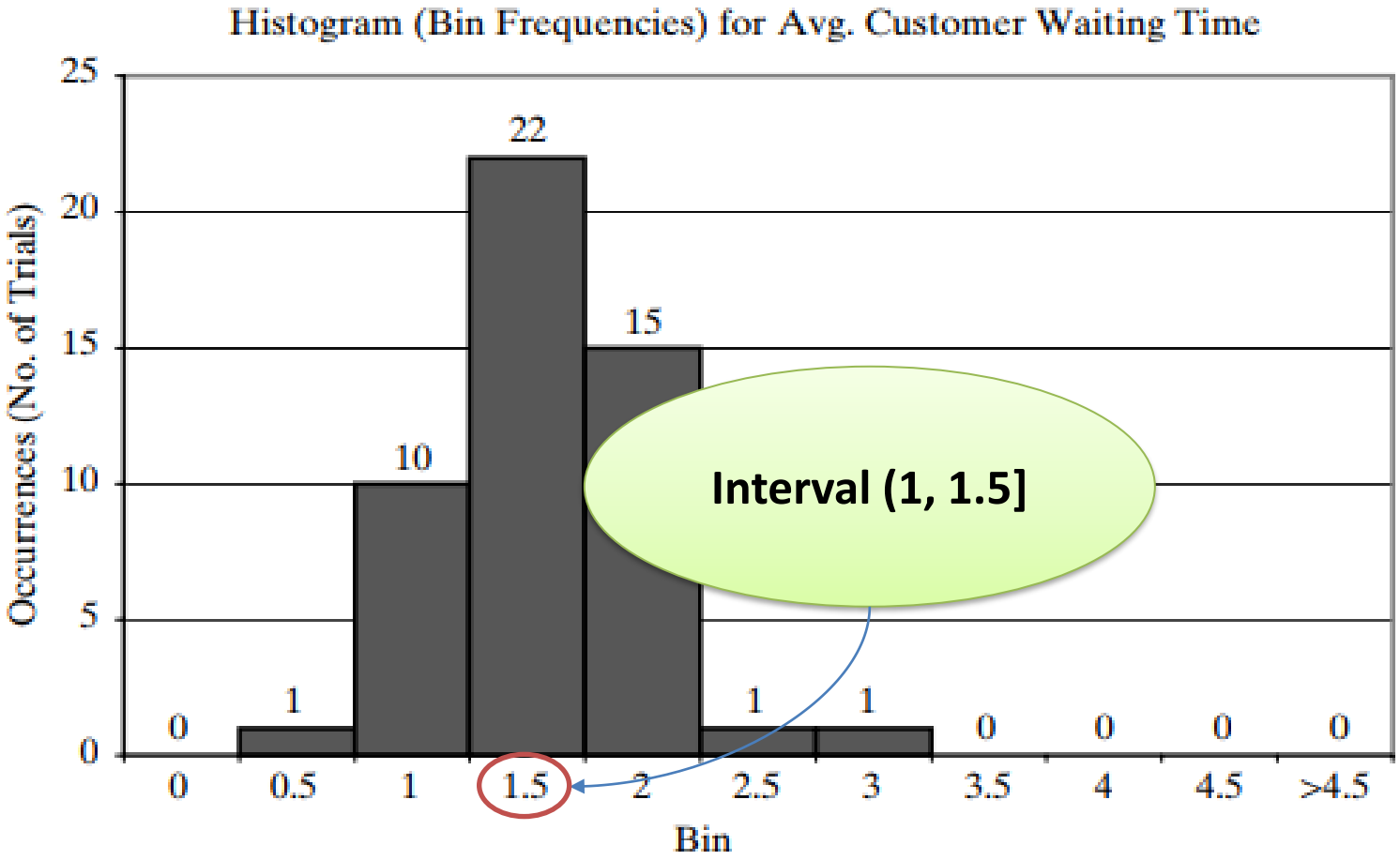
Sim. a Single-Server Queue (14/14)

Some of the findings from the simulation are as follows (4/4):



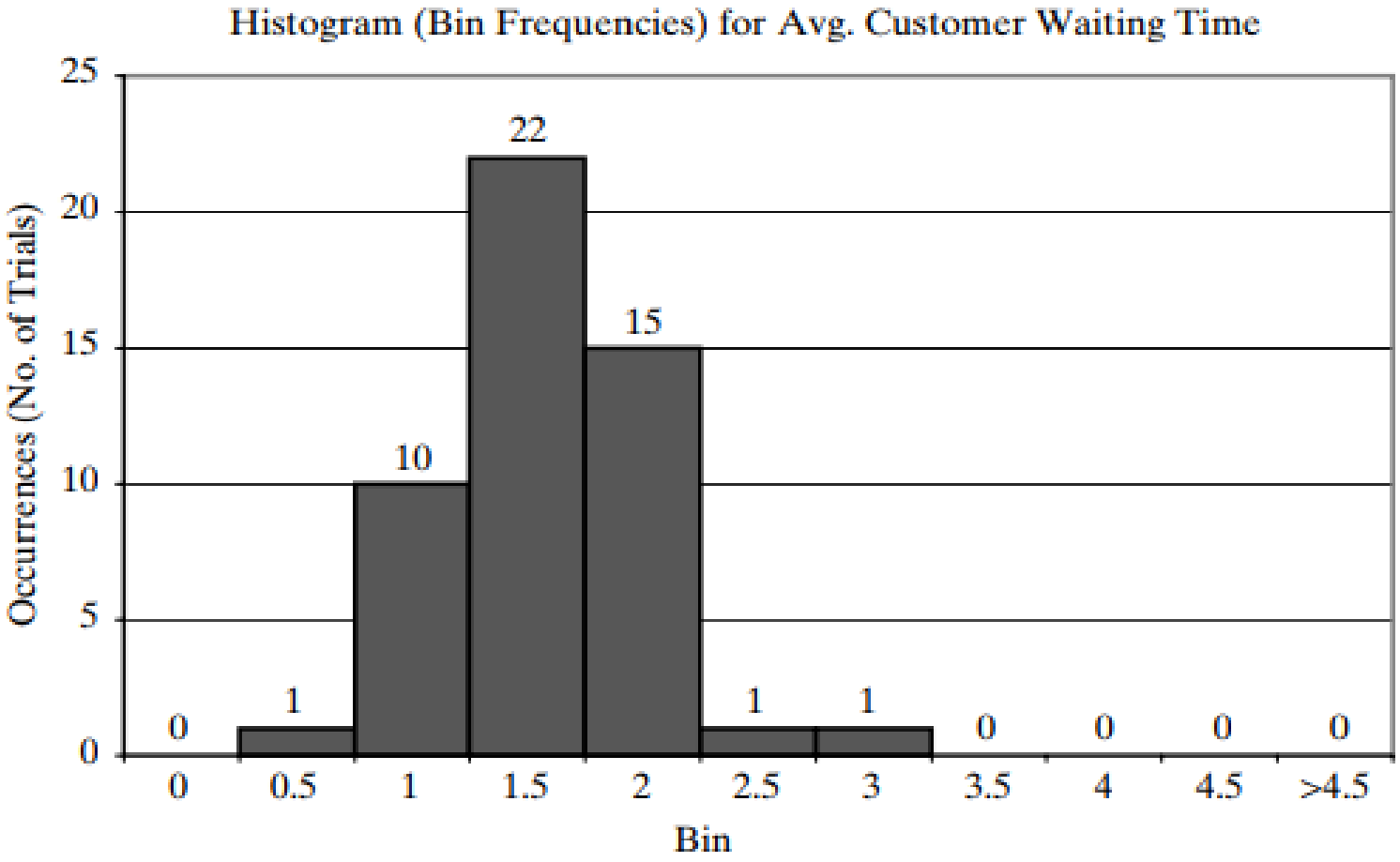
Sim. a Single-Server Queue (14/14)

Some of the findings from the simulation are as follows (4/4):



Sim. a Single-Server Queue (14/14)

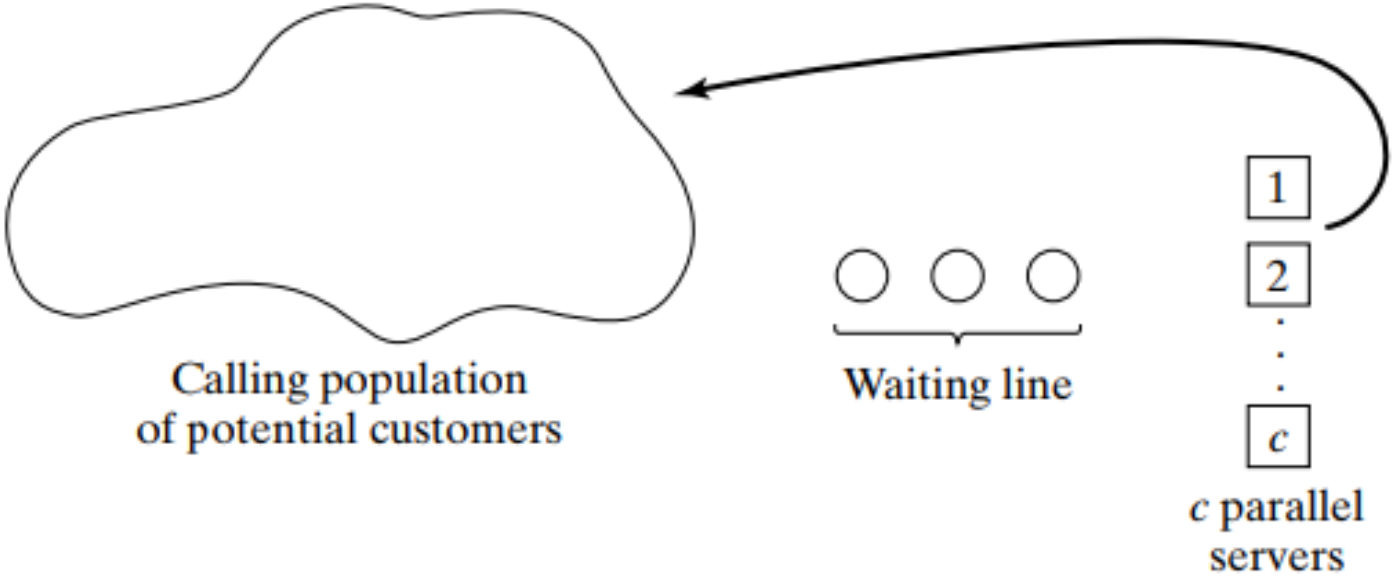
Some of the findings from the simulation are as follows (4/4):



Sim. a Multi-Server Queue (1/18)

Queueing Model:

- Suppose that there are *c* channels operating in parallel. Arrivals will join a single queue and enter the first available service channel.



Sim. a Multi-Server Queue (2/18)

Example: Call Center:

Sim. a Multi-Server Queue (2/18)

Example: Call Center:

- Consider a Call Center where technical personnel take calls and provide service.
- Two technical support people (**2 server**) exists:
  - ***Able*** more experienced, provides service faster,
  - ***Baker*** newbie, provides service slower.
- **Rule:**
  - *Able* gets call if both people are idle.

Sim. a Multi-Server Queue (3/18)

Interarrival distribution of calls for technical support

Time between Arrivals (Minutes)	*Probability*
1	0.25
2	0.40
3	0.20
4	0.15

Sim. a Multi-Server Queue (3/18)

Interarrival distribution of calls for technical support

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*
1	0.25	0.25
2	0.40	0.65
3	0.20	0.85
4	0.15	1.00

Sim. a Multi-Server Queue (3/18)

Interarrival distribution of calls for technical support

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.25	0.25	01–25
2	0.40	0.65	26–65
3	0.20	0.85	66–85
4	0.15	1.00	86–00

Sim. a Multi-Server Queue (4/18)

Service Distribution of Able

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Service Distribution of Baker

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Sim. a Multi-Server Queue (5/18)

The simulation:

- The problem is to find how well the current arrangement is working. To estimate the system measures of performance, a simulation of **1 hour** of operation is made. A longer simulation would yield more reliable results, but for purposes of illustration a l-hour period has been selected.

Sim. a Multi-Server Queue (6/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			Baker			
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
1											
2											
3											
4											
5											

Customer = Call

Sim. a Multi-Server Queue (6/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			Baker			
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
1											
2											
3											
4											
5											

Interarrival Time

Sim. a Multi-Server Queue (6/18)

Simulation Table:

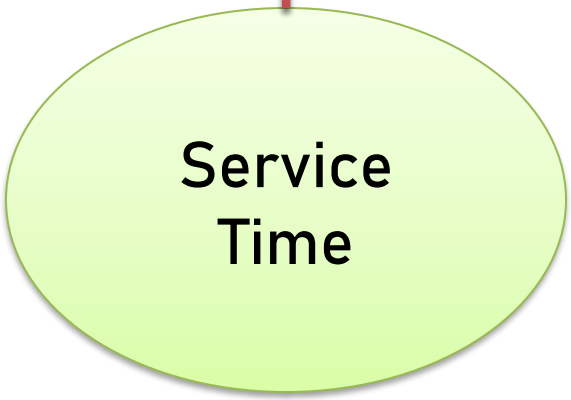
A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1											
2											
3											
4											
5											

Time of Arrival

Sim. a Multi-Server Queue (6/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			Baker			
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
1											
2											
3											
4											
5											



Sim. a Multi-Server Queue (6/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			Baker			
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
1											
2											
3											
4											
5											

Waiting Time

Sim. a Multi-Server Queue (7/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			Baker			
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
1	–	–	0	95							
2											
3											
4											
5											

Sim. a Multi-Server Queue (7/18)

Simulation Table:

A	B	C	D	E
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*
1	–	–	0	95
2				
3				
4				
5				

Service Time (Minutes)	**Able** *Probability*	*Cumulative Probability*	*Random-Digit Assignment*
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Service Time (Minutes)	**Baker** *Probability*	*Cumulative Probability*	*Random-Digit Assignment*
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Sim. a Multi-Server Queue (7/18)

Simulation Table:

A	B	C	D	E
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*
1	–	–	0	95
2				
3				
4				
5				

Service Time (Minutes)	**Able** *Probability*	*Cumulative Probability*	*Random-Digit Assignment*
2	0.30	0.30	01–30
3	0.28	0.58	31–58
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3	0.35	0.35	01–35
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Sim. a Multi-Server Queue (7/18)

Simulation Table:

A	B	C	D	E
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service
1	–	–	0	95
2				
3				
4				
5				

Able

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Baker

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Sim. a Multi-Server Queue (7/18)

Simulation Table:

A	B	C	D	E
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service
1	–	–	0	95
2				
3				
4				
5				

Able

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Baker

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Sim. a Multi-Server Queue (7/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2											
3											
4											
5											

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26										
3											
4											
5											

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C
Customer No.	Random Digits for Arrival	Time between Arrivals
1	–	–
2	26	
3		
4		
5		

Time between Arrivals (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
1	0.25	0.25	01–25
2	0.40	0.65	26–65
3	0.20	0.85	66–85
4	0.15	1.00	86–00

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C
Customer No.	Random Digits for Arrival	Time between Arrivals
1	–	–
2	26	
3		
4		
5		

Time between Arrivals (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
1	0.25	0.25	01–25
2	0.40	0.65	26–65
3	0.20	0.85	66–85
4	0.15	1.00	86–00

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2									
3											
4											
5											

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2								
3											
4											
5											

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21							
3											
4											
5											

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C	D	E
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service
1	–	–	0	95
2	26	2	2	21
3				
4				
5				

Able

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Baker

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C	D	E
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service
1	–	–	0	95
2	26	2	2	21
3				
4				
5				

Able

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Baker

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C	D	E
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service
1	–	–	0	95
2	26	2	2	21
3				
4				
5				

Able

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Baker

Service Time (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Sim. a Multi-Server Queue (8/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3											
4											
5											

Sim. a Multi-Server Queue (9/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			Baker			
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4											
5											

Sim. a Multi-Server Queue (10/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5											

Sim. a Multi-Server Queue (11/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			*Baker*			
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0

Sim. a Multi-Server Queue (12/18)

Simulation Table:

A	B	C	D	E	F	G	H	I	J	K	L
					Able			Baker			
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1

Sim. a Multi-Server Queue (13/18)

Simulation Table (62 minutes)

A	B	C	D	E	F	G	H	I	J	K	L
						Able			*Baker*		
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	61	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	01				32	3	35	1
15	34	2	33	81	35	4	39				2
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	01	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11

Sim. a Multi-Server Queue (13/18)

Simulation Table (62 minutes)

A	B	C	D	E	F	G	H	I	J	K	L
						Able			*Baker*		
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	61	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	01				32	3	35	1
15	34	2	33	81	35	4	39				2
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	01	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11

Sim. a Multi-Server Queue (13/18)

Simulation Table (62 minutes)

A	B	C	D	E	F	G	H	I	J	K	L
						Able			Baker		
Customer No.	Random Digits for Arrival	Time between Arrivals	Clock Time of Arrival	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
1	—	—	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2									0
6	42	2									1
7	74	3									1
8	80	3									0
9	68	3									0
10	22	1									0
11	48	2									1
12	34	2									0
13	45	2									0
14	24	1									1
15	34	2									2
16	63	2									0
17	38	2									2
18	80	3									0
19	42	2									1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11

Time between Arrivals (Minutes)	Probability	Cumulative Probability	Random-Digit Assignment
1	0.25	0.25	01–25
2	0.40	0.65	26–65
3	0.20	0.85	66–85
4	0.15	1.00	86–00

Sim. a Multi-Server Queue (13/18)

Simulation Table (62 minutes)

Service Time (Minutes)	Able Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Service Time (Minutes)	Baker Probability	Cumulative Probability	Random-Digit Assignment
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

A	B	C	D	E	F	G	H	I	J	K	L
						Able			Baker		
Customer	Random Digits	Time between	Clock Time	Random Digits for Service	Time Service Begins	Service Time	Time Service Ends	Time Service Begins	Service Time	Time Service Ends	Time in Queue
11	48	2	26	95	0	5	5				0
12	34	2	28	21				2	3	5	0
13	45	2	30	51	6	3	9				0
14	24	1	31	92	10	5	15				0
15	34	2	33	89				12	6	18	0
				38	15	3	18				1
				13	18	2	20				1
				61	20	4	24				0
				50				23	4	27	0
				49	24	3	27				0
				39	27	3	30				1
				53				28	4	32	0
				88	30	5	35				0
				01				32	3	35	1
				81	35	4	39				2
				53				35	4	39	0
				81	39	4	43				2
				64				40	5	45	0
				01	43	2	45				1
				67	45	4	49				1
				01				48	3	51	0
				47	49	3	52				0
				75				51	5	56	0
				57	54	3	57				0
				87				56	6	62	1
				47	59	3	62				0
						56			43		11

Sim. a Multi-Server Queue (13/18)

Simulation Table (62 minutes)

A	B	C	D	E	F	G	H	I	J	K	L
						Able			*Baker*		
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	61	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	01				32	3	35	1
15	34	2	33	81	35	4	39				2
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	01	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11

Sim. a Multi-Server Queue (14/18)

The analysis of simulation table results in the following:

1. Over the 62-minute period Able was busy 90% of the time.
2. Baker was busy only 69% of the time. The seniority rule keeps Baker less busy (and gives Able more tips).
3. Nine of the 26 arrivals (about 35%) had to wait. The average waiting time for all customers was only about 0.42 minute (25 seconds), which is very small.

Sim. a Multi-Server Queue (15/18)

The analysis of simulation table results in the following:

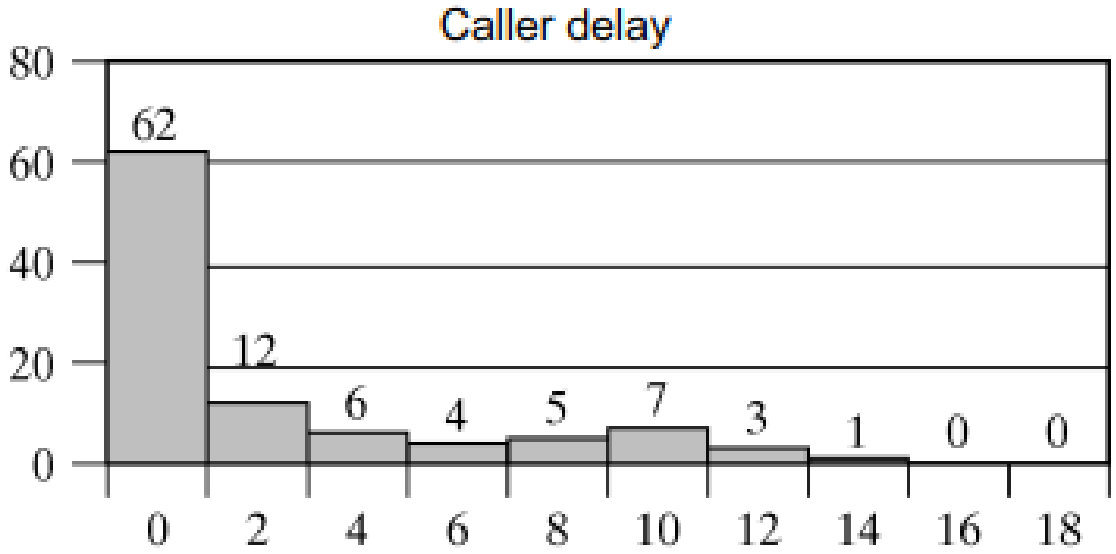
4. Those nine who did have to wait only waited an average of 1.22 minutes, which is quite low.

5. In summary, this system seems well balanced. One server cannot handle all the calls, and three servers would probably be too many. Adding an additional server would surely reduce the waiting time to nearly zero. However, the cost of waiting would have to be quite high to justify an additional server.

Sim. a Multi-Server Queue (16/18)

Simulation run for 100 calls:

- 62% of callers had no delay.
- 12% of callers had a delay up to 2 minutes.

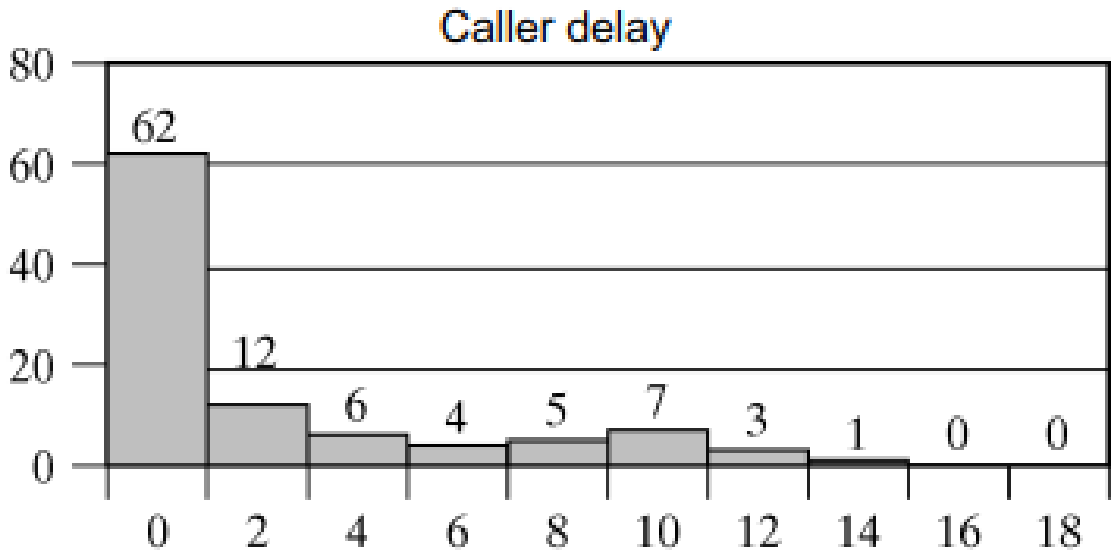


Sim. a Multi-Server Queue (16/18)

Simulation run for 100 calls:

- 62% of callers had no delay.
- 12% of callers had a delay up to 2 minutes.

For example:
Average waiting time
= 2 minutes



Sim. a Multi-Server Queue (17/18)

400 simulation trials of 100 caller:

1. Average waiting time = 2 minutes.
2. Average waiting time = 1.7 minutes.
3. Average waiting time = 1.9 minutes.
4. Average waiting time = 2.5 minutes.
5. Average waiting time = 1.5 minutes.

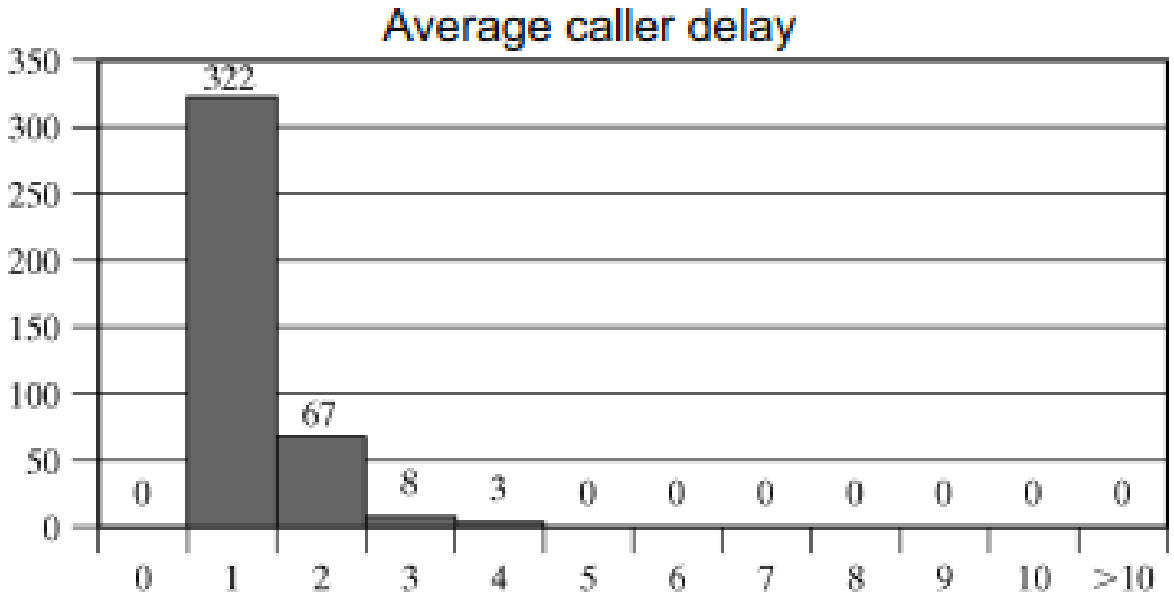
…

400. Average waiting time = 2.7 minutes.

Sim. a Multi-Server Queue (18/18)

400 simulation trials of 100 caller:

- 80.5% of callers had delay up to 1 minute.
- 19.5% of callers had delay more than 1 minute.



Assignment #1 – (2 Marks)

رابط تحميل السؤال	https://drive.google.com/drive/folders/114O9ZS2cqQCXPFOYgRiFjNUwclvcOZcZ?usp=share_link
بداية ظهور السؤال	السبت 11 مارس 2023
رابط ارسال الاجابة	https://forms.gle/jfyxgj6qRGx6JevVA
اخر موعد لارسال الاجابة	الخميس 23 مارس 2023 الساعة 11:59 م

Assignment #1 – (2 Marks)

Submission Instructions

Each student will write a combined report for their work that has the following:

1. You must submit **one** file with all answers in one report with a cover page that includes: course code, course name, academic year, semester, instructor, assignment #, student name, department, and email, etc.
2. The file must be named **SC311-A1-IS-StudentName.pdf**
   Where "**IS**" is for "Information Systems department". Write the name of your department (IS or SC), and **StudentName** should be your **Name**.
3. The file must be in either **MS Word** format or in **PDF** format.
4. Upload the file in the following link: https://forms.gle/jfyxgj6qRGx6JevVA
5. Make sure you test this process of uploading in advance before the deadline.

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij

Lecture #4: https://www.youtube.com/watch?v=h65-6EBO-pg&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=13

https://www.youtube.com/watch?v=enKpt5KgZAE&list=PLxIvc-MGOs6geFJmdvDOIN5zE89-Hq8Ij&index=14

Thank You

Dr. Ahmed Hagag

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